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Prediction of urban residential end-use water demands by integrating known and unknown water demand drivers at multiple scales II: Model application and validation

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ABSTRACT

Detailed prediction of end-use water demand at multiple spatial and temporal scales is essential for planning urban water supply using multiple water sources based on fit-for-purpose criteria. This paper presents the application of a stochastic model to predict urban residential end-use water demands at multiple spatial and temporal scales. The model includes an improved representation of spatial and temporal variability of urban residential water use by considering the effect of a significant number of water demand drivers such as household size, dwelling type, appliance efficiency, availability of water end-uses/appliances at dwellings, presence of children, presence of people at home, diurnal behavioral patterns and temperature. A stochastic approach is used to describe the variability of residential water demand that is not captured by these known explanatory variables. The model is validated against quarterly meter readings and hourly water use data. The validation of household water demand at a quarterly scale with billing data shows Correlation coefficients (R^2) ranging between 90% and 96% and Nash-Sutcliffe coefficients ranging between 0.70 and 0.92 for the four seasons analyzed which, verifies the predictive capacity of the model. The model validation also demonstrates the statistical stability of the selected probability distributions used in modeling the unexplained behavior of urban residential water consumers. The hourly scale validation also demonstrates a satisfactory predictive capacity in predicting household water demand. This also evidences the effectiveness of the modeling approach to predict urban residential water demand at multiple temporal scales.

1. Introduction

Integrated Urban Water Management (IUWM) is an approach that has emerged in recent years to address the problem of water scarcity with sustainable solutions. Understanding water demand by end-use at multiple temporal and spatial scales is a key consideration for implementing IUWM. This understanding is essential for matching water demands to sources of appropriate quality and quantity (fit-for-purpose water supply) in which water supply sources such as rooftop water, stormwater and recycled water are available at various spatial scales (i.e. dwelling, development, city) and temporal scales (i.e. sub-daily, daily, seasonal).

Detailed prediction of water demand requires the ability to describe demand responses to changes such as demographics, household characteristics and weather (Kenney et al., 2008; House-Peters et al., 2010; Makki et al., 2015). Rathnayaka et al. (2014) studied the variables explaining the significant variability in household water use and found that the variables household size, typology of dwelling, appliance efficiency, presence of children under 12 years of age, presence of end-uses/water use appliances can explain about 40% of the variability among households. Further, the same authors highlighted the importance of representing this residual and unknown variability which can be attributed to behavioral factors such as culture and personal preferences among water users in demand modeling. Findings confirm that socio-economic factors including ethnicity, education and income affect water use

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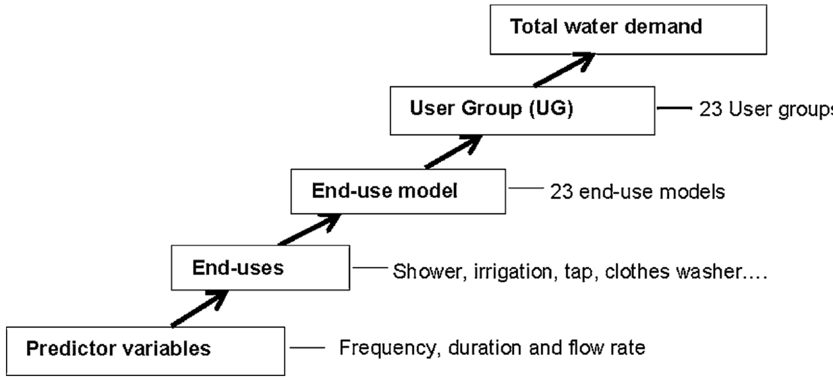

Fig. 1. Key components of the model.

behavior (Makki et al., 2015; Garcia-Cuerva et al., 2016). However, unavailability of such data prevented these factors being studied in detail. Rathnayaka et al. (2015) also studied the temporal variability of water end-uses at multiple scales to understand dynamics of household water use over time showing that end-uses have statistically significant differences between seasons. In a later study, Rathnayaka et al. (forthcoming) also found that water demand of some end-uses vary between weekdays to weekends. The same authors conducted an extensive literature review that revealed the lack of a model that integrates these complex water demand dynamics and their underlying variables into a single framework that predicts end-use water demand at multiple scales, especially at small scales, explores "what if" scenarios to support decision making and has a robust explanatory capacity to support IUWM and planning.

In order to represent these complex water demand dynamics at multiple spatial and temporal scales, Rathnayaka et al. (forthcoming) have developed an improved model to predict urban residential water demand. This paper presents the application of this modeling method to predict water end-use demand and the validation of the model against household water use. Section 2 briefly describes the modeling method while Section 3 presents the application of the modeling method to water end-uses. Finally, the validation of the model at different temporal scales is presented in Section 4 followed by the conclusion.

2. The urban residential end-use water demand model

The urban residential end-use water demand model applied in this study is presented in detail in a companion paper by Rathnayaka et al. (forthcoming) and briefly summarized herein. The model consists of several steps where different variables are considered providing the capacity to predict end-use water demands at multiple spatial and temporal scales. Fig. 1 shows the key components of the model.

The model consists of 23 User Groups (UG) based on household size, dwelling type and presence of children less than 12 years of age. This is a unique feature of this model, which also considers the presence of end-uses and efficient appliances in households and behavioral differences between these different types of users. Unique end-use models developed for each user group describe the unknown variability within each user group. The end-use models predict demand of shower, toilet, tap, bath, dishwasher, clothes washer, evaporative cooler and garden irrigation water end-uses. Each end-use is modelled using predictor variables in which probability distributions are used to represent the unknown behavior of water consumers. In addition, distinct probability distributions are used to represent altered water use behaviors due to weather, duration of people at home or appliance efficiency.

3. Application of the modeling concept to water end-uses

The application of the model to different end-uses is described here in detail followed by the validation of the aggregate result of all the end-use models. The predictive capacity of individual end-use water demand models could not be validated against end-use data due to the lack of end-use data.

Household water demand is obtained by summing predicted end-use water demands as described by Rathnayaka et al. (forthcoming). End-use water demand is predicted by considering whether the end-use is present in the household. The respective volume of water demand is determined as described in detail next (Rathnayaka et al., forthcoming).

3.1. Shower water demand model

The volume of shower water demand is predicted considering appliance efficiency, season and behavior of people. The penetration rate of efficient showers for each UG is considered in modeling shower water demand at UG level while, shower water demand per day per household is predicted using three predictor variables: frequency, duration and flow rate (Eq. (1)):

$$WD_s = Fr \times Dur \times G \tag{1}$$

Where "WD_{S}" is shower water demand per household per day, "Fr" is the frequency of shower use per household per day, "Dur" is the duration (Min) of a shower event and "G" is the flow rate (L/min) of a shower event. Random values for shower frequency are generated using a specifically identified probability distribution. If shower frequency is greater than zero, then random numbers are generated for flow rate and duration according to the identified Probability Distribution Functions (PDF).

General regression analysis (Freund et al., 2006) was used to determine the degree of correlation between predictor variables. From this analysis, frequency, flow rate and duration are found to be independent variables. Therefore, the volume of shower water demand can then calculated by the product of the three predictor variables.

Shower water use in winter shows a statistically significant difference from summer (Rathnayaka et al., 2014). Therefore, shower water demand for the two seasons is modelled separately using probability distributions representative of frequency, flow rate and duration variables for the season. The data used in identifying these probability distributions, and methods used in selecting the best-fit probability distribution are discussed by Rathnayaka et al. (forthcoming). The specific probability distributions that have been shown to describe each user group in winter are shown in Table 1.

The Poisson distribution fits shower frequency data for most user groups particularly the user groups with larger household

Table 1
Probability distributions selected to describe shower water demand predictors in winter.

User group	Shower frequency	Shower duration	Shower flow rate –Efficient	Shower flow rate-Standard
1	Binomial (n = 1,p = 0.70256)	GenExtreme(k = 0.1676,σ = 1.4092,μ = 4.1966)	Nakagami (m = 1.4832,Ω = 58.059)	Wakeby (4.1244, 0.23201,0,0,15.811)
2	Binomial (1,0.70256)	GenExtreme(0.1676,1.4092,4.1966)		
3	Poisson (λ = 0.68132)	GenExtreme (0.18581,2.1292,3.9193)		
4	Binomial(27,0.02564)	JohnsonSB(γ = −0.08409,δ = 0.69693,λ = 6.9417,ζ = 1.5865)		
5	Binomial(27,0.02564)	JohnsonSB(−0.08409,0.69693,6.9417,1.5865)		
6	Poisson (2.0784)	JohnsonSB(2.2753,1.5152,21.367,1.5586)		
7	Poisson (2.3077)	JohnsonSB(1.3907,1.1428,13.179,2.1243)		
8	Binomial (13,0.13762)	JohnsonSB(2.9253,1.5111,48.691,0.15906)		
9	Poisson (1.8097)	JohnsonSB(2.8292,1.4843,47.207,0.17956)		
10	Poisson (2.6)	Triangular(m = 5.0833,a = 3.5288,b = 11.047)		
11	Poisson (2.6)	Triangular(5.0833,3.5288,11.047)		
12	Poisson (2.6)	Triangular(5.0833,3.5288,11.047)		
13	Poisson (2.6)	Triangular(5.0833,3.5288,11.047)		
14	Poisson (2.3333)	LogPearson3 (α = 13.201,β = −0.10329,γ = 2.9232)		
15	Poisson (2.3172)	GenExtreme (0.13408,2.3988,6.0437)		
16	Poisson (2.6538)	GenLogistic (k = −0.0432,σ = 1.1263,μ = 7.0638)		
17	Poisson (2.6538)	GenLogistic (−0.0432,1.1263,7.0638)		
18	Poisson (2.1963)	GenLogistic(0.21149,1.5326,5.7424)		
19	Poisson (3.6093)	Dagum(k = 0.76588, α = 4.2196, β = 7.1042, γ = 1)		
20	Poisson (2.4103)	Wakeby (α = 16.149,β = 7.8386,γ = 3.9213,δ = 0.20712,ζ = 1.7041)		
21	Poisson (4.2436)	Wakeby(11.402,6.4897,1.4209,−0.08345,3.8139)		
22	Poisson (3.7451)	Dagum(0.82294,4.6915,5.5979,1)		
23	Poisson (4.1538)	GenLogistic(0.02646,0.7182,7.5854)		

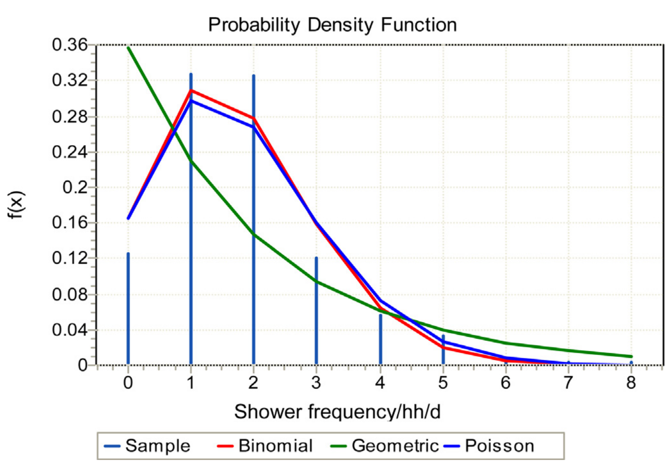

Fig. 2. Probability distributions fit into shower frequency per household (hh) per day (d).

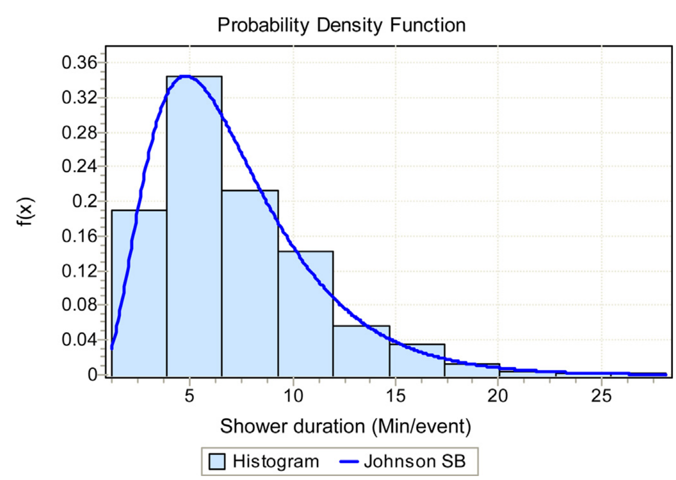

Fig. 3. Best-fit probability distribution for shower duration-UG 9, Winter (min/event).

sizes while the Binomial distribution fits shower frequency data in rest of the user groups (Table 1). Binomial distribution is used to explain integer events where number of trials seems fixed compared to Poisson distribution that explains events that seem to take place over and over again in a completely haphazard way (Wolfram, 2016). For illustration purpose, Fig. 2 displays the histogram obtained for winter shower frequency data of UG 9 (Household size 2, detached house with no children) and it shows that the Binomial, Geometric and Poisson probability distributions fit to the observed data set.

Based on Anderson-Darling (A-D) test and histograms, the Poisson distribution is shown to be the best-fit PDF to describe shower frequency for UG 9.

Probability distributions such as the Burr distribution and Log-logistic distribution which commonly fit the winter shower duration data for all user groups although it does not represent the best-fit probability distribution for each user group. Depending on distribution parameters, the Burr distribution and Log-logistic distribution may be unimodal with a single "peak" or monotone decreasing with a potential singularity approaching the lower boundary of its domain (Wolfram, 2016). In addition, these probability distributions have tails that are "fat" in the sense that its PDF decreases algebraically rather than exponentially for large values (Wolfram, 2016). For example, the best-fit probability distribution to describe winter shower duration for UG 9 is the Johnson SB distribution as shown in Fig. 3. Johnson SB distribution is unimodal with a single "peak", though its overall shape (its height, its spread, and its concentration near the axis) is completely determined by the values of its arguments and it has "thin" tale in the sense that the PDF decreases exponentially rather than decreasing algebraically for large values of x (Wolfram, 2016). Accordingly, Johnson SB distribution explains the shower duration of UG 9 (Fig. 3).

Further, distinct probability distributions are identified for modeling efficient and standard shower flow rates (Table 1). Standard showers use 15 to 25 L/min specified in the Water efficiency labelling and standards (Australian Government, 2013) although flow rates can be smaller depending on user adjustment. Using this criterion, shower flow rates of standard showers greater than 15 L/min occurring just once during the measurements are grouped

Table 2
Descriptive statistics for flow rates of efficient and standard showers.

Descriptive statistics	Efficient showers	Standard showers
Mean (L/min)	6.99	19.16
Coefficient of Variation	0.43	0.14
Sample Variance	9.17	7.36
Range	15.79	9.07
Minimum (L/Min)	0.17	16.13
Maximum (L/Min)	15.96	25.20
Count	5651.00	157.00

separately from the water efficient showers that use less than 15 L/min. Table 2 shows the descriptive statistics for flow rates of efficient and standard showers and their variability.

The Nakagami distribution closely fits the flow rates (L/min) of efficient showers while the Wakeby distribution fits the flow rate data from standard showers in winter (Table 1). These same probability distributions fit the summer data albeit with different distribution parameters (Table 3).

3.2. Toilet water demand model

Frequency of toilet water use and flush volume are used to predict toilet water demand as shown in Eq. (2). Alternative probability distributions are used to describe dual and single flush volumes with their specific penetration rates for each efficiency type at user group level (Rathnayaka et al., Forthcoming).

$$WD_{Toi} = Fr \times Vol \tag{2}$$

where "WD_{Toi}" is water demand for toilet use in liters per household per day, "Fr" is number of toilet use events per household per day and "Vol" is flush volume in liters per event.

Toilet use frequency is determined stochastically using the Negative Binomial and Poisson probability distributions as listed in Table 4.

A value for toilet flush volume is generated when the random number generated for frequency of toilet use is greater than zero. Toilet flush volume varies according to their efficiency type (Rathnayaka et al., 2014). Households with toilet flush volume 9 and 11 L/event are classified as single flush toilets while households with toilet flush volume 4.5/3, 6/3, 9/4.5, 11/6 L/event are classified as dual flush toilets. Descriptive statistics for single and dual flush volumes show the significant difference and the variability of flush volume for two types of toilets (Table 5). These statistical differences demonstrate the importance of using specific probability distributions to describe this predictor variable based on efficiency type. Comparatively few data points are available for single flush toilets which explains the weak visual fit of the probability distribution with the observed data for single flush toilets (Fig. 4).

The Gen-extreme value and Nakagami distributions show the best-fit for dual and single flush volumes, respectively, which are then applied based on the percentage of dual and single flush toilets in each user group.

3.3. Tap water demand model

Tap water use includes bathroom, laundry and kitchen water use. Tap water demand is predicted from frequency of tap use and volume of tap use (Eq. (3)):

$$WD_{Tap} = Fr \times Vol \tag{3}$$

where "WD_{Tap}" is tap water demand in liters per household per day, "Fr" is events of tap use per household per day and "Vol" is event volume of tap use in liters.

Frequency values for tap use are first generated and if they are greater than zero, a stochastic value for tap volume is generated to calculate the volume of tap use. Table 6 summarizes the selected best-fit probability distributions to describe tap frequency and volume for each user group. Fig. 5 shows a typical example of the probability distributions fit for frequency data of UG 14. It can be observed that the Negative Binomial is the best-fit distribution for UG 14 as shown by the application of the A-D test.

Table 3
Probability distributions selected to describe shower water demand predictors in summer.

User group	Shower frequency	Shower duration	Shower flow rate –Efficient	Shower flow rate-Standard
1	Poisson(0.95349)	Dagum(0.56122[a],2.2302,4.161,1)	Nakagami (1.4313,55.631)	Wakeby (21.188,28.002,3. 2996,0.05917,14.51)
2	Poisson(0.95349)	Dagum(0.56122,2.2302,4.161,1)		
3	Poisson(0.525)	Wakeby(375.72,153.33,4.1619,−0.16611,0)		
4	Poisson(1)	JohnsonSB(0.79363,0.53421,13.058,3.9338)		
5	Poisson(1)	JohnsonSB(0.79363,0.53421,13.058,3.9338)		
6	Poisson(2.25)	JohnsonSB(0.47119,1.3037,11.984,0.75463)		
7	Poisson(2.6957)	Wakeby(9.2562,3.2141,1.984,−0.22502,2.1686)		
8	Poisson(2.1598)	GenExtreme(0.09075,1.9927,4.8056)		
9	Poisson(2.1825)	GenExtrem(0.08839,2.0142,4.8111)		
10	Poisson(2.1957)	Weibull(α = 1.3905,β = 3.4792,γ = 3.273)		
11	Poisson(2.1957)	Weibull(1.3905,3.4792,3.273)		
12	Poisson(2.1957)	Weibull(1.3905,3.4792,3.273)		
13	Poisson(2.1957)	Weibull(1.3905,3.4792,3.273)		
14	Poisson(2.6555)	Wakeby(7.0838,3.5141,1.4049,−0.07539,2.2074)		
15	Poisson(2.9799)	Weibull(1.5864,5.2206,2.0924)		
16	Poisson(2.2727)	GenLogistic(0.3001,1.6897,7.0214)		
17	Poisson(2.2727)	GenLogistic(0.3001,1.6897,7.0214)		
18	Poisson(2.8652)	Gaussian(χ = 29.866,σ = 5.3224,μ = 0.21316)		
19	Poisson(3.9919)	Burr(k = 1.1058,α = 4.0642,β = 5.4761,γ = 1.0151)		
20	Poisson(3.2873)	InvGaussian(χ = 24.988, μ = 6.6184,λ = 0.29174))		
21	Poisson(3.1591)	GenExtreme(0.30093,1.3403,5.3609)		
22	Poisson(4.2754)	Dagum(0.9349,4.0642,4.8931,1)		
23	Poisson(2.6)	Wakeby(242.31,58.219,3.0675,−0.4029,0)		

[a] Sensitivity of these decimals are considerable.

Table 4
Probability distributions selected to describe predictor variables of toilet water demand in all user groups.

User group	Toilet frequency	Volume of toilet use-Dual flush toilets	Volume of toilet use-Single flush toilets
1	NegBinomial(n = 9,p = 0.58266)	GenExtreme value(0.0111, 1.881, 4.5242)	Nakagami (2.2743, 97.134)
2	NegBinomial(9,0.58266)		
3	Poisson(4.2067)		
4	Poisson(7.825)		
5	Poisson(7.825)		
6	NegBinomial(5,0.38353)		
7	NegBinomial(6,0.40528)		
8	NegBinomial(3,0.26754)		
9	NegBinomial(3,0.27352)		
10	NegBinomial(6,0.38251)		
11	NegBinomial(6,0.38251)		
12	NegBinomial(6,0.38251)		
13	NegBinomial(6,0.38251)		
14	NegBinomial(3,0.27937)		
15	NegBinomial(3,0.23443)		
16	Poisson(9.9077)		
17	Poisson(9.9077)		
18	NegBinomial(5,0.3275)		
19	NegBinomial(3,0.19162)		
20	NegBinomial(6,0.3443)		
21	NegBinomial(5,0.24745)		
22	NegBinomial(9,0.35135)		
23	NegBinomial(5,0.2393)		

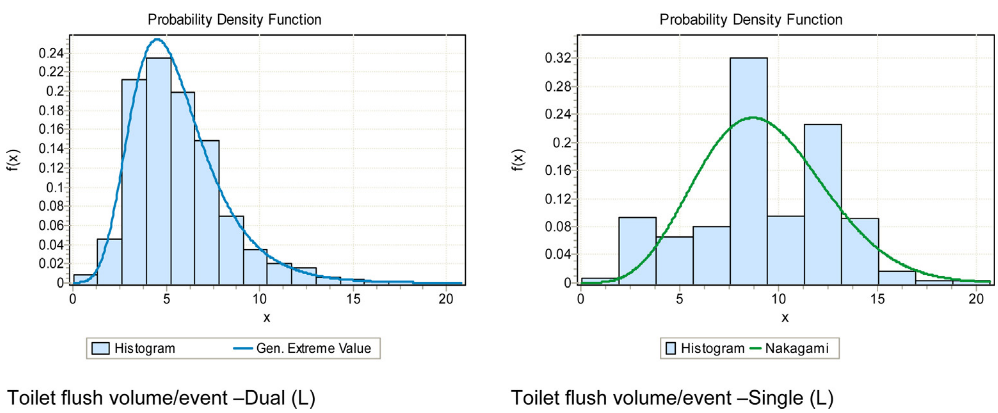

Fig. 4. Probability distribution fits for observed toilet flush volume data (L/event).

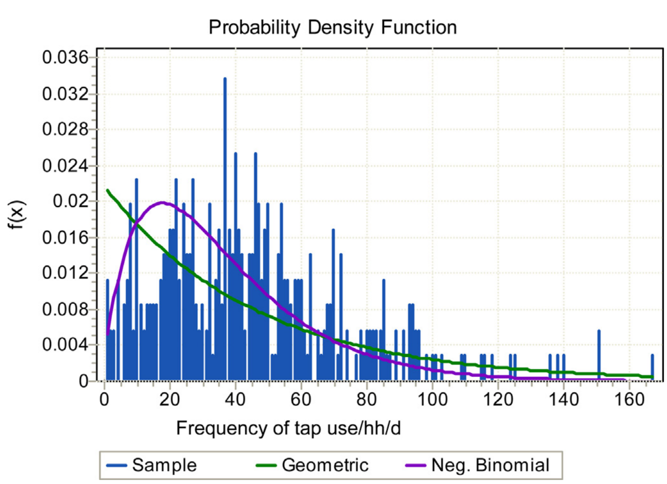

Fig. 5. Best-fit probability distributions for tap frequency for UG 14.

3.4. Bath water demand model

Bath water demand is predicted by the occurrence of bath use and volume of bath water use variables (Eq. (4)). Considering the fact that half of the households with bath do not use bath (Rathnayaka et al., forthcoming), its volume is scaled by the number of households in each user group and by half the bath penetration rate in each user group to predict the bath water demand of each user group.

Table 5
Descriptive statistics for single and dual flush volumes.

Descriptive Statistics	*Single flush toilets*	*Dual flush toilets*
Mean (L)	9.24	5.63
Coefficient of Variation	0.37	0.43
Sample Variance	11.68	5.98
Range	20.67	20.79
Minimum (L)	0.04	0.03
Maximum (L)	20.71	20.82
Sample size	1484	47446

$$WD_{Bath} = Occur \times Vol \quad (4)$$

where "WD_{bath}" is bath water demand in liters per household per day, "Occur" is a binary variables (1,0) that represents whether household occupants use the bath within the day and "Vol" is volume of bath water use in liters per household per day.

Values for occurrence of bath use are generated randomly using the best-fit probability distribution. If bath use has occurred, a value for bath volume is then generated. The bath water use data was not grouped into user groups due to the small number of data points.

Table 6
Probability distributions selected to describe frequency and volume variables of tap water demand for all user groups.

User group	Tap frequency	Tap volume/event
1	Negative binomial (n = 7,p = 0.32862)	GenExtreme(k = 0.19457,σ = 0.35975,μ = 0.77299)
2	Negative binomial (7,0.32862)	GenExtreme0.19457,0.35975,0.77299)
3	NegBinomial(3,0.07126)	LogPearson3(α = 67.484,β = −0.06824,γ = 4.3405)
4	NegBinomial(3,0.08095)	Wakeby(α = 24.227,β = 23.557,γ = 0.30506,δ = −0.14727,ζ = −0.06012)
5	NegBinomial(3,0.08095)	Wakeby(24.227,23.557,0.30506,−0.14727,−0.06012)
6	Geometric(p = 0.0222)	Wakeby(30.692,71.016,0.91058,−0.28526,0)
7	Geometric(0.01957)	GenExtreme(−0.03479,0.47299,0.9963)
8	NegBinomial(2,0.03952)	LogPearson3(137.31,0.04787,−6.5344)
9	NegBinomial(2,0.03977)	LogPearson3(144.93,0.04675,−6.7338)
10	NegBinomial(5,0.08592)	GenExtreme(0.1985,0.24683,0.69766)
11	NegBinomial(5,0.08592)	GenExtreme(0.1985,0.24683,0.69766)
12	NegBinomial(5,0.08592)	GenExtreme(0.1985,0.24683,0.69766)
13	NegBinomial(5,0.08592)	GenExtreme(0.1985,0.24683,0.69766)
14	NegBinomial(2,0.05232)	GenLogistic(0.33891,0.43888,1.2896)
15	Geometric(0.02133)	GenExtreme(0.32658,0.45664,1.0407)
16	NegBinomial(3,0.04246)	LogPearson3(35.385,0.06461,−2.4449)
17	NegBinomial(3,0.04246)	LogPearson3(35.385,0.06461,−2.4449)
18	NegBinomial(3,0.05381)	GenExtreme(0.11261,0.38155,0.84423)
19	Geometric(0.01308)	LogPearson3(28.118,0.09957,−2.6569)
20	Geometric(0.01284)	GenLogistic(0.25963,0.25224,1.0148)
21	NegBinomial(6,0.09215)	Wakeby(1.8277,4.029,0.53135,−0.14942,0.47637)
22	NegBinomial(3,0.03239)	LogPearson3(91.757,−0.04687,4.375)
23	NegBinomial(3,0.03162)	GenExtreme(−0.09279,0.24918,1.0763)

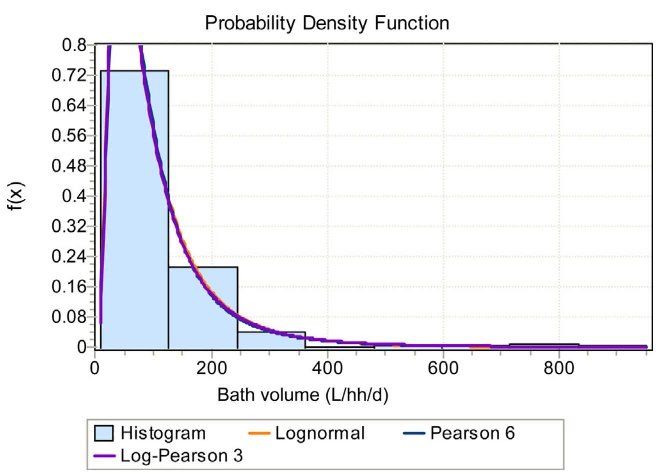

Fig. 6. Probability distributions fit for bath volume.

This led us to identify the best-fit probability distribution for bath occurrence and volume that are common for all residential users. The Binomial distribution is the best-fit for occurrence of bath use that generates binary values. The Log-normal distribution is the best-fit probability distribution to describe the volume of bath use according to the A-D test, albeit the Pearson 6 and Log-Pearson 3 also fit bath volume data equally well (Fig. 6).

3.5. Dishwasher water demand model

Dishwasher water demand is predicted using the occurrence and volume predictor variables (Eq. (5)). To predict dishwasher water demand, data is generated randomly for occurrence of a dishwasher event using the identified best-fit probability distribution. If the event has occurred, then the volume of dishwasher water use is generated in a similar fashion to occurrence:

$$WD_{Dish} = Occur \times Vol. \tag{5}$$

where "WD $_{\text{Dish}}$" is the dishwasher water demand in liters per household per day, "Occur" is a binary variable (0,1) that shows whether the household operates the dishwasher on the day and "Vol" is dishwasher water use in liters per household per day.

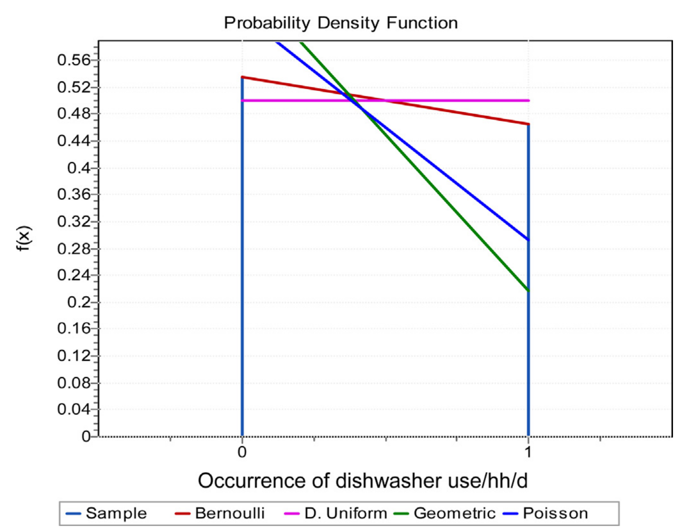

Fig. 7. Probability distributions fit for occurrence of dishwasher use for UG 12 data.

The percentage of dishwasher ownership or the penetration rate is applied to each user group when the model is used to predict dishwasher water demand of a group of households. About 30% of the households in CWW (City West Water) and YVW (Yarra Valley Water) data samples do not have a dishwasher at home. Therefore, the amount of data to model dishwasher water demand of some user groups is rather limited. Several user groups from UG 1 to 8 have no data while some of them have less than 5 records. To overcome this limitation, it is assumed that households of size 1 and 2 behave similarly for dishwasher use and the combined data is used to model dishwasher water demand for UGs 1 to 8.

The Bernoulli distribution fits the data of dishwasher use occurrence for all UGs albeit with different probability distribution parameters. Table 7 presents the probability distributions selected to describe occurrence and volume variables of dishwasher water demand for each user group.

Other probability distributions such as Poisson, Geometric and Uniform distributions also fit the occurrence data of dishwasher use (Fig. 7). These probability distributions together with observed data of UG 12 are displayed in Fig. 7.

Table 7
Probability distributions selected to describe occurrence and volume variables of dishwasher water demand for each user group.

User group	Occurrence of dishwasher	Dishwasher volume (L/day)
1to 8	Bernoulli(p = 0.11111)	Wakeby(33.063,3.6211,1.7957,0.29939,1.9337)
9	Bernoulli(0.302)	GenLogistic(0.18561,3.6817,13.868)
10	Bernoulli(0.46429)	Wakeby(139.75,9.233,2.5128,0.33236,0.37138)
11	Bernoulli(0.46429)	Wakeby(139.75,9.233,2.5128,0.33236,0.37138)
12	Bernoulli(0.46429)	Wakeby(139.75,9.233,2.5128,0.33236,0.37138)
13	Bernoulli(0.46429)	Wakeby(139.75,9.233,2.5128,0.33236,0.37138)
14	Bernoulli(0.31481)	Wakeby(48.111,7.4228,5.9416,0.10569,4.6363)
15	Bernoulli(0.41226)	GenExtreme(0.08354,5.9746,14.568)
16	Bernoulli(0.69231)	Wakeby(195.7,32.61,22,−1.0934,0)
17	Bernoulli(0.69231)	Wakeby(195.7,32.61,22,−1.0934,0)
18	Bernoulli(0.4507)	Wakeby(39.011,4.6321,3.0666,0.44524,2.1005)
19	Bernoulli(0.37161)	Wakeby(78.14,15.014,13.99,−0.16433,1.7366)
20	Bernoulli(0.49049)	GenGamma(1.0179,2.735,4.7865,1)
21	Bernoulli(0.45872)	Wakeby(53.893,3.9658,0.56794,0.33346,1.0849)
22 to 24	Bernoulli(0.43966)	GenLogistic(0.23234,5.31,15.078)

3.6. Clothes washer water demand model

Clothes washer water demand is predicted using occurrence and volume predictor variables (Eq. (6)):

$$WD_{cloth=Occur\times Vol} \tag{6}$$

where "WD_{cloth}" is water demand for clothes washer per household per day, "Occur" is a binary variable (0,1) that describes whether the clothes washer is operated on the day, and "Vol" is the volume of water used by the clothes washer per household per day.

Since clothes washer water use shows a statistically significant difference between weekdays and weekends (Rathnayaka et al., forthcoming), the occurrence of clothes washer use is modelled distinctively for weekends and weekdays by alternative probability distributions. If the clothes washer is operated, then a value for volume of clothes washer water use is generated randomly using the respective probability distribution. The volume is determined using distinctive probability distributions identified for top and front loader clothes washers and they are used based on the respective penetration rates in each user group (Rathnayaka et al., forthcoming). This grouping reduces the data available for model clothes washer water use for each user group level due to the limited amount of data available for common water end-uses such as clothes washer, dishwasher, evaporative cooler and irrigation compared to everyday water end-uses. To overcome this limitation, data from similar household sizes were aggregated to have sufficient aggregate data that allows the development of separate models for each household size.

Table 8 shows the best fit probability distributions for each probability space of the clothes washer water demand model. The Bernoulli distribution shows the best-fit for occurrence of clothes washer use events, both for weekends and weekdays albeit with different distribution parameters for different household sizes (Table 8).

3.7. Garden irrigation water demand model

The literature identifies maximum daily temperature, occurrence and magnitude of rainfall events as the main determinants of irrigation water demand (Duncan and Mitchell, 2008; Maidment and Miaou, 1985, 1986). Rathnayaka et al. (2014) found that garden size and irrigation equipment also explain summer water use of households with gardens. In light of these findings, it is also important to study the effect of alternative water supplies such as grey water and rainwater on irrigation water demand. However, lack of sufficient data precluded this factor to be included in this study. Instead, we take an alternative temperature-determined stochastic model approach based on actual observations to describe irrigation water demand.

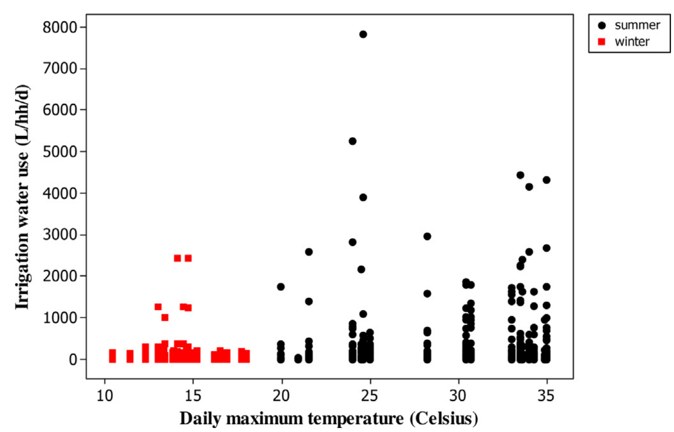

Fig. 8. Daily irrigation water use of individual households vs. maximum daily temperature (Celsius).

This model considers penetration rate of gardens and irrigation volume based on three predictor variables – occurrence, duration and flow rate – thus increasing the adaptability of the model to suit the significant variability among individual households in the use of alternative irrigation methods:

$$WD_{Irri} = Occur \times G \times Dur \tag{7}$$

Where "WD_{Irri}" is the irrigation water demand per household per day, "Occur" is a binary variable (0, 1) that describes whether occupants irrigate their garden on the day, "G" is flow rate (L/min) and "Dur" is total duration of irrigation water use (min/d). Fig. 8 shows a scatter plot between daily irrigation water use data at household scale and daily maximum temperature.

The variability of irrigation water use in summer above 20 °C is significantly large among individual households for the same temperature while this variability is smaller in winter. Both the CWW and YVW end-use data show that water is used for garden irrigation also during winter. The data sample contain 225 daily records of winter irrigation events (9% of total records) showing that irrigation water use in winter is similar in magnitude to dishwasher and bath water use.

Further analysis was carried out using two sample *t*-tests to understand the difference in irrigation water use variables between winter and summer and the results are shown in Table 9.

Although the three variables show a statistically significant difference between the two seasons, only the duration and occurrence variables show a considerable difference in magnitude compared to their average values.

Table 8
Probability distributions selected to describe occurrence and volume variables of clothes washer demand.

User group	Occurrence of clothes washer event/day		Volume of clothes washer use(L/day)	
	weekend	Week day	Front loader	Top loader
1to 3	Bernoulli (0.23265)	Bernoulli (0.20606)	JohnsonSB(γ = 0.89007,δ = 0.77908, μ = 152.37,σ = 22.925)	Burr (0.69356,4.863,137.24,1)
4 to 9	Bernoulli (0.25349)	Bernoulli (0.20849)	GenLogistic(0.2091,25.215,63.201)	Dagum(0.78806,2.597,166.26,1)
10 to 15	Bernoulli (0.56207)	Bernoulli (0.42747)	Wakeby (2163.2,75.327,65.007,0.43496,0)	Wakeby(2072.9,28.999,82.368,0.26005,0)
16 to 19	Bernoulli (0.57929)	Bernoulli (0.49275)	GenExtreme(0.34278,40.321,67.974)	Wakeby(1207.1,15.052,104.78,0.12064,−8.1815)
20 and 21	Bernoulli (0.5848)	Bernoulli (0.49864)	FatigueLife (α = 0.72467,β = 88.124,γ = 1)	Frechet (α = 2.0291,β = 167.24,γ = 1)
22 and 24	Bernoulli (0.58929)	Bernoulli (0.64655)	LogPearson3(17.104,−0.14425,6.8264)	Wakeby(309.98,2.7917,75.766,0.33174,−6.0009)

Table 9
Results of two-sample *t*-test of irrigation water use variables between winter and summer.

Variable	Mean difference	T value	P value	DF
Occurrence/household/day	0.153	7.17	0.000	30
Duration/day	1688 s (28 min)	3.75	0.001	27
Flow rate/event	1.558 L/min	2.54	0.011	552

Table 10
Difference in irrigation water use between detached and other dwellings.

Variables	Detached houses	Flat/unit/ semidetached houses
Average volume (L/hh/d)	53.7 (75.2 in summer, 16.3 in winter)	25.5 (40.7 in summer, 3.1 in winter)
Average occurrence/day –Summer	0.2	0.3
Average duration (Min/day) –Summer	43.9	11.7
Average flow rate (L/min) –Summer	11.5	11.4

Garden size varies between individual households, but such detailed data is not commonly available. However, garden size is found to be a major causal factor governing the difference in household water use between detached dwellings and other dwellings with gardens (Rathnayaka et al., 2014). This difference in irrigation water use between dwellings was further studied using actual end-use data.

Table 10 shows that all variables except flow rate are significantly different between detached and other dwellings, in particular, the duration of irrigation water use that can be considered to represent the effect of garden size. Based on these observations, the volume of irrigation water use is predicted for detached and other dwellings by distinctly different probability distributions for duration and occurrence (Eq. (7)).

To describe each predictor variable, the best-fit probability distributions selected for irrigation water demand are shown in Table 11. Two probability distributions are identified for temperatures below and above 20 °C. If occupants irrigate their gardens, values are randomly generated for duration and flow rate variables. While the probability distribution for the duration variable is also based on temperature, the probability distribution for flow rate is not temperature dependant (Table 11). It is important to note that the probability distribution for flow rate described herein represents the flow rates of hoses fitted with a trigger nozzle.

Lack of sufficient independent water end-use datasets to validate end-use models is still a limitation in studies of water end-use. This is particularly relevant to irrigation end-use. Unlike other end-uses, irrigation is an end-use that shows a significant temporal variability. In this study, we were compelled to use data limited to two-week periods one from winter and another from summer. This can be considered borderline to develop a model that represents all four seasons characterized by different rainfall patterns, rainfall amounts, sunlight and temperatures. Notably, the amount of rainfall that occurred during the end-use measurements is lower (0.4 mm/day) than the annual average rainfall for that period (1.5 mm/day) which may not be sufficiently significant to affect people's decision on garden irrigation. Hence, collecting water end-use data representative of all the four seasons and for longer time periods to support modeling of irrigation water demand is a high priority.

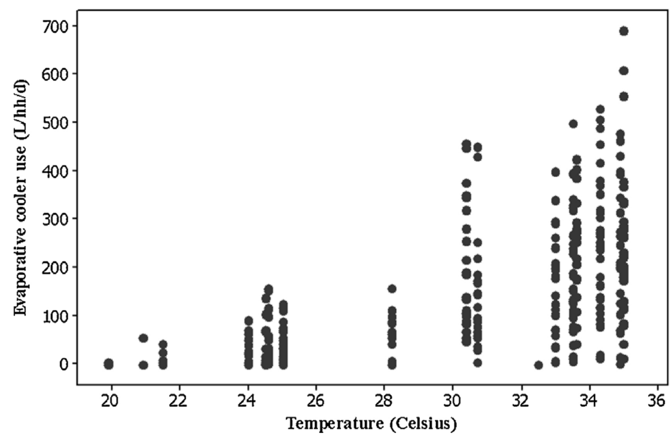

Fig. 9. Scatter plot of maximum daily temperature and volume of evaporative cooler use (L/hh/d).

3.8. Water demand model for evaporative cooler

Evaporative coolers in residential water use may only consume 10% of summer water use, but its peak water demand at a daily and hourly scale can be significantly high, up to 336 L/hh/d on a hot day. This attribute makes it important to consider the prediction of their water demand at small scale.

Athuraliya et al. (2012) shows that maximum daily temperature is a key determinant of evaporative cooler water use. Our data for water use for individual households for the 2012 summer is shown in the scatter plot of maximum daily temperature and volume of evaporative cooler use (Fig. 9)

In Fig. 9, it can be observed that the volume of evaporative cooler water use, its variability, and the number of households operating evaporative coolers increase with increasing temperature. The evaporative cooler use datasets for 2005 and 2012 are

Table 11
Probability distributions and their parameters for irrigation water demand model.

Variables	Model for detached dwellings		Model for other dwelling types	
	If daily maximum temperature is above or similar to 20 °C	If daily maximum temperature is below 20 °C	If daily maximum temperature is above or similar to 20 °C	If daily maximum temperature is below 20 °C
Occurrence	Bernoulli (0.23106)	Bernoulli (0.12954)	Bernoulli (0.29677)	Bernoulli (0.06731)
Duration	PhasedBiWeibull (α1 = 1.0385,β1 = 18.612, γ1 = 0,α2 = 0.68194, β2 = 25.107,γ2 = 10.5)	Wakeby (−4.5182,0.47891, 9.1527,0.432,0.85407)	PhasedBiWeibull (1.0979,7.1764,0, 0.73869,10.419,3.3333)	LogPearson3 (1.8119,0.63149,−0.11988)
Flow rate Wakeby (19.17,3.5302,1.1377,−0.08021,5.5835)				

Table 12
Percentage occurrence of evaporative cooler events by temperature range.

Temperature (T) range °C	20 = < T < 25	25 = < T < 30	30 = < T < 35	T = > 35
Occurrence% in 2005	19.6%	50.7%	66.7%	71.9%
Occurrence% in 2012	31.7%	38.5%	72.8%	No data

Table 13
Probability distribution and their parameters for occurrence of evaporative cooler events.

Temperature range	20 = < T < 25 °C	25 = < T < 30 °C	30 < T < 35 °C
Probability distribution	Bernoulli	Bernoulli	Bernoulli
Distribution parameter (p)	0.34677	0.46457	0.75758

used to observe these relationships, while the summer 2012 end-use dataset is used to develop the model to predict evaporative cooler water demand.

Evaporative cooler water demand model considers penetration rate of evaporative coolers in user groups, occurrence of use (hh/d) and volume of evaporative cooler water use (hh/d). Since only 25% of households in the data sample had evaporative coolers, end-use data from all user groups were combined to develop this demand model. This approach ignores the difference in water use behavior between users from different user groups.

The percentage of households that operated evaporative coolers during the measurement periods in 2005 and 2012 for different temperature levels was determined from observations of evaporative cooler use (Table 12). The percentage of households that operate evaporative coolers increases with increasing temperature, although it must be observed that a considerable number of households do not use their evaporative coolers at any temperature level (Table 12). It is important, however, to note that about 30% of households do not operate evaporative coolers even at over 30 °C (Table 12). This may be due to believing evaporative coolers are ineffective at high temperatures or the presence of other cooling methods or residents being absent from the house during day time.

Considering this variability in triggering evaporative coolers and its relationship with temperature (Table 12; Fig. 9), a temperature dependent probability distribution is employed to model the occurrence of this event. This approach explains water use behavior of households triggered by temperature. The threshold trigger temperature level employed in this model is 20 °C. A range of discrete probability distributions including Bernoulli, Binomial, Discrete Uniform, Geometric, and Poisson were fitted to the observed values of occurrence for each temperature level. The Bernoulli distribution with a binary variable was identified as the best-fit according to the A-D goodness of fit test (Yap and Sim, 2011). Table 13 lists the distribution parameters for the Bernoulli distribution for each temperature range.

Eq. (6) is applied to describe the occurrence of an event applying the temperature-determined stochastic approach using an "IF" logical condition with the probability distribution parameters shown in Table 13.

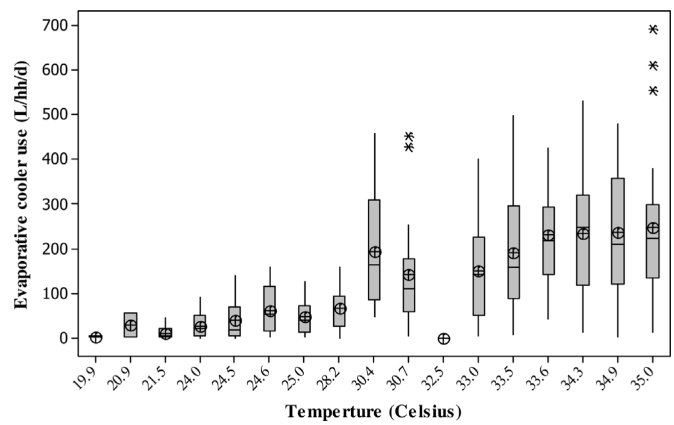

Fig. 10. Box plot between maximum daily temperature and evaporative cooler water use (L/hh/d) observed from 2012 evaporative cooler water use data.

$$\text{Occurrence} = \text{IF}(T < 20, 0, \text{IF}(T < 25, \text{BernoulliRand}(0.34677), \text{IF}(T < 30, \text{BarnoulliRand}(0.46457), \text{BernoulliRand}(0.75758)))) \quad (6)$$

where T denotes that maximum daily temperature in degree Celsius.

The relationship between volume of evaporative cooler water use and maximum daily temperature is shown in Fig. 8 using a box plot. In this figure, a clear difference in water use behavior below and above 30 °C can be observed. The average evaporative cooler use and its variability are greater for temperatures above 30 °C (Fig. 10). To reflect this temperature boundary, the volume of evaporative cooler water use is predicted stochastically using alternative probability distributions to describe the evaporative cooler water use for these two temperature ranges (Eq. (7)).

A Wakeby distribution is identified as the best-fit probability distribution to explain the volume of evaporative cooler water use. Table 14 shows the distribution parameters for two identified temperature ranges.

Volumes of evaporative cooler water demand are generated randomly by applying Eq. (7) based on deterministic daily temperature data.

Table 14
Distribution parameters for Wakeby distribution for two temperature ranges.

Temperature range		20 = < T < 30 °C	30 = < T °C
Distribution parameters	α	59.362	290.02
	β	0.1557	2.661
	γ	0	149.6
	δ	0	−0.1745
	ξ	−7.6532	−5.3473

Table 15
Key characteristics of the data samples.

Data sample	Sample 1 (100 households)	Sample 2 (1131 households)
Average household size	3.05	3.00
Dwelling composition	Flat-3%, Semi-detached-11%, Detached-86%	Flat-11%, Semi-detached-9%, Detached-79%
Presence of children under 12 years	With children-29%, With no children-71%	With children-31%, with no children-69%

$$\mathrm{Ev} = \mathrm{IF}(T < 20, 0, \mathrm{IF}(\mathrm{AND}((T < 30)), (\mathrm{WakebyRand}(59.362, 0.15567, 0, 0, -7.6532)), (\mathrm{WadebyRand}(290.02, 2.661, 149.6, -0.174478, -5.3473)))) \quad (7)$$

where Ev denotes the volume of evaporative cooler water use per household per day and T is the temperature in °C.

The predicted evaporative cooler water demand for a particular user group (EU_e) is determined by multiplying the volume of household evaporative cooler water demand (Ev) when it is operated, by the number of households with evaporative coolers in a user group as follows:

$$EU_e = (N \times PR\%) \times (IF\,(Occurence > 0)\,, (E_v)\,, (0)) \quad (8)$$

where N denotes number of households in the user group and PR is the associated penetration rate for evaporative coolers in the user group.

4. Model validation

The model was validated with household water use data at quarterly and hourly time scales. The quarterly scale validation was carried out using two sets of household water use data from 2010/12 YVW Residential End-Use Measurement study (Sample 1) and YVW 2011 Appliance Stock and Usage Pattern (ASUP) web-based survey (Sample 2). Key characteristics of the data samples used in quarterly scale validation are given in Table 15. While Sample 2 consists of smaller average household size and more flats than Sample 1, data samples are more alike in presence of children.

4.1. Quarterly scale model validation

The first sample of meter readings (Sample 1) was obtained from 100 households that participated in the 2010/12 YVW REUM study. On the other hand, the period of validation extends between the winter 2009 and the autumn 2012, a period that is significantly longer than each of the two-week periods from the winter 2010 and summer 2012 from which end-use data was collected and used in the model development. Weather data from Melbourne Regional Office weather station (086071) was used for prediction of demand. The change in penetration rates of appliance efficiency and presence of water use appliances at households for the periods of validation is negligible and hence is assumed constant. For example, the stock change for uptake of efficient showers is 1% during the period of validation (Integrated resource planning for urban water, 2011). The average of quarterly water demand (L/hh/d) predicted

Table 16
R^2 and Nash-Sutcliffe coefficients for observed and predicted average water use at user groups level.

Quarter	R^2	Nash-Sutcliffe coefficient
Winter 2009	0.75	0.64
Spring 2009	0.69	0.54
Summer 2010	0.68	0.48
Autumn 2010	0.82	0.79
Winter 2010	0.84	0.79
Spring 2010	0.73	0.72
Summer 2011	0.78	0.42
Autumn 2011	0.79	0.67
Winter 2011	0.72	0.70
Spring 2011	0.69	0.51
Summer 2012	0.63	0.49
Autumn 2012	0.57	0.43

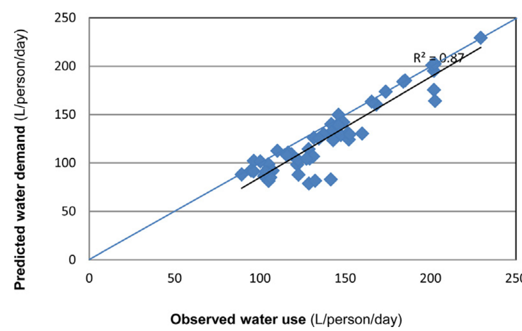

Fig. 11. Scatter plot between observed and predicted water use at UG level.

for each user group was validated against the average observed data prepared using quarterly meter readings. As validation metrics, Correlation coefficients (R^2) and Nash-Sutcliffe model efficiency coefficients (Vieux, 2004) were estimated for each quarter using observed and predicted average water use for user groups as shown in Table 16.

The model shows greater predictive capacity (i.e. R^2 and Nash-Sutcliffe coefficient ≈ 0.75) in some quarters while in other quarters the predictive capacity is moderate (i.e. R^2 and Nash-Sutcliffe coefficient ≈ 0.50) (Table 16). User groups with fewer observations show a poor performance between observed and predicted data. The percentage differences between predicted annual average water use (L/hh/d) and observed annual average water use (L/hh/d) across all user groups are 13.1%, 11.6% and 12.7% for years 2009/10, 2010/11 and 2011/12, respectively.

The second sample of data (Sample 2) was obtained from 1241 households that participated in the YVW 2011 ASUP web-based survey. The meter readings from summer 2011 to spring 2011 are used in this analysis. This analysis allowed validation of the model with an independent set of data from a more representative sample of YVW customers. The scatter of the results shown in Fig. 11 demonstrates that the model slightly underestimates the prediction of water demands for user groups. Roberts et al. (2011) show that their end-use data measurement sample has more low users and fewer high water users compared to their customer population which results in a 15% difference in average water use. This is a factor that can affect any data collection since low water users may be proud of their water usage and be willing to participate in surveys and measurements (Quilliam, 2012). The R^2 and Nash-Sutcliffe Coefficient are estimated as 0.87 and 0.68, respectively, between the predicted and observed daily water demands at UG level.

The R^2 and Nash-Sutcliffe coefficients show a significantly high predictive capacity for the model in all four quarters for all tested user groups (Table 17). Furthermore, the model shows 88% accuracy

Table 17
R^2 and Nash-Sutcliffe coefficient for summer 2011 to spring 2011.

Quarter	R^2	Nash-Sutcliffe coefficient
Winter 2011	0.94	0.73
Spring 2011	0.95	0.77
Summer 2011	0.98	0.95
Autumn 2011	0.95	0.85

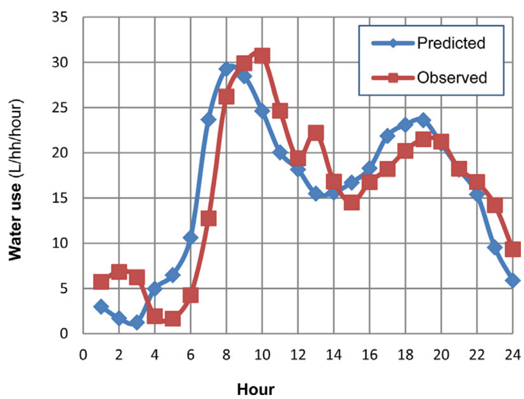

Fig. 12. Observed and predicted average hourly water use (summer 2011).

in predicting the annual average water use for residential water demand.

The model validation carried out at a quarterly scale demonstrates that the model can predict residential water demand with high accuracy. It also demonstrates the statistical stability of probability distributions used in modeling "unknown behavior" observed during two-week periods, one in winter 2010 and the other in summer 2012.

4.2. Model validation at an hourly scale

The model development is based on end-use data collected from the REUM 2010/12-YVW sample during summer 2012. Conversely, the hourly validation of the model relies on an independent data observed at hourly scale collected during summer 2011 (1st of December 2010 to 29th of February 2011). For the purpose of model validation, summer water demand was predicted using weather data from the same period. This average daily water demand for summer 2011 was disaggregated based on the percentage hourly household water use pattern for summer (Athuraliya et al., 2012). The model predicted hourly water demand for an average summer day in 2011 is shown in Fig. 12 together with the observed data used to validate the model.

A Nash-Sutcliffe coefficient of 0.73 and R-square of 0.75 further confirm the model's validity at an hourly scale. The overall predictive capacity of the model for hourly scale is 85%. This comparison reveals that the model prediction has a good fit with the observed data. A slight shift in the predicted peak to the left is observed in the first peak while the slight under estimation of the peak demand are well within the expected capability of this type of model, and provides sufficient confidence in the use of the model for the prediction of water demand variations.

5. Conclusions

Prediction of water end-uses at multiple spatial and temporal scales is essential for planning and management of decentralized water supply systems. This paper presents the application of a prediction model with aforesaid capacity to different water end-uses and its validation at different temporal scales.

The model considers the effect of a large number of significant water demand drivers such as household size, dwelling type, appliance efficiency, availability of water end-uses/appliances at dwellings, presence of children, temperature, presence of people at home, and diurnal behavioral patterns affecting different spatial and temporal scales. In addition, the effect of user behavior that cannot be explained using these known water demand drivers is incorporated into the model stochastically. A range of probability distributions were tested and identified the best-fit probability distributions to describe individual probability spaces for each predictor variable defined by factors such as efficiency, season and presence of people which could affect its behavior.

The model was validated against two sets of household data at quarterly and hourly scale. Validation of household water demand at a quarterly scale was carried out with utility billing data yielding a R^2 ranging between 90% and 96% and Nash-Sutcliffe coefficient ranging between 0.70 and 0.92 for the four seasons analyzed. Hourly scale validation also demonstrated a satisfactory predictive capacity ($R^2 = 0.75$). The model validation demonstrates the statistical stability of the probability distributions used in modeling the unexplained behavior of urban residential water consumers and also the ability of the model to predict urban residential water demand at multiple temporal scales.

The study recommends collecting more independent water end-use datasets to validate end-use models which is still a limitation of this study while collecting irrigation water use data representative of all the four seasons and for longer time periods to support modeling of irrigation water demand is a high priority.

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